

Teaching reform exploration & practice: probability & mathematical statistics

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Abstract: Based on teaching practice, this paper discusses the teaching reform of Probability Theory and Mathematical Statistics from aspects of stimulating students' learning interest, combining blackboard writing with multimedia teaching, adopting heuristic teaching, and using software-assisted teaching, aiming to improve learning effects and cultivate students' ability to analyze and solve problems.

Keywords: Probability Theory and Mathematical Statistics, teaching methods, teaching reform, ability.

1. Introduction

1.1. Overview and significance of probability theory and mathematical statistics

Probability Theory and Mathematical Statistics is a branch of mathematics that studies and reveals the statistical regularity of a large number of random phenomena, and deduces and induces statistical laws. It is one of the public basic courses for undergraduates and postgraduates of most science, engineering, management and other disciplines in institutions of higher education.

It not only has a rigorous theoretical system but also strong practicality. Probability theory contains modern mathematical ideas such as limit, differential and integral, integrates tools like analysis and algebra, and provides a theoretical basis for data processing. Mathematical statistics, based on probability theory, is a discipline that studies how to effectively collect, sort out and analyze random data, and then make inferences and decisions on concerned issues. It is widely used in industrial and agricultural production, science and technology, and various departments of the national economy.

Therefore, this course is very beneficial for cultivating college students' mathematical literacy and ability to solve practical problems.

1.2. Learning difficulties and course reform suggestions

Different from other mathematical branches, probability statistics is the first random mathematics that beginners come into contact with.

Due to the abstract nature of the knowledge, students generally reflect that it is difficult to understand the concepts, or they can understand the course but are at a loss when encountering problems, or mechanically apply formulas by referring to examples without knowing the reasons, or shrink back because they need to use advanced mathematics knowledge.

Based on the phenomenon that students have difficulty in developing thinking and mastering methods in learning, combined with the characteristics of the course and teaching practice, this paper puts forward the following suggestions for the course reform.

2. Stimulating students' learning interest

2.1. Stimulating prob & stats interest with examples

As the saying goes, interest is the best teacher. The same is true for learning Probability and Statistics. Only by mobilizing students' learning enthusiasm and arousing their interest can they draw inferences

from one instance and achieve twice the result with half the effort. Otherwise, abstract conceptual knowledge is likely to arouse students' fear, and the teaching effect will be half-hearted.

Like other mathematics courses, the knowledge system of probability statistics is complex, rigorous and relatively abstract, which is easy to make students feel boring and lose interest in learning probability statistics, thus fostering weariness of learning.

Therefore, how to arouse students' curiosity about this course and promote their active learning and exploration has become the primary task of teachers.

Teachers need to catch students' attention, arouse their interest and leave a good impression in the first class. Compared with other mathematics courses, probability statistics is often easier to achieve this.

Probability statistics itself originates from life and is used to solve specific problems in life. Teachers can introduce problems through some current and down-to-earth examples.

For example, there are a large number of detective films and TV dramas currently, in which the protagonists often say something like this: what is seen now is just an accident or coincidence, but a series of coincidences are no longer coincidences. This sentence can be understood from a statistical perspective. We can calculate the probability of the success of this event and the probability of making mistakes when rejecting it. Teachers can put forward this problem for students to think about, and wait until they finish learning hypothesis testing to think about and understand the principles.

In addition, teachers can also mention how to understand divination and constellations from the perspective of probability statistics, understand life creeds, and strengthen the principles of dealing with affairs, such as "don't ask for divination when there's nothing wrong, but you can refer to it when there's something wrong".

2.2. Probability origin & fun interactive teaching

When introducing the origin of probability, teachers can introduce its background, namely the problem of fair distribution of stakes.

Two gamblers agreed to bet 5 games, and the one who wins 3 games first wins; now A has won 2 games and B has won 1 game. Due to some reasons, the gambling was terminated. How should the stakes be distributed reasonably? We can think as follows: there are four possible outcomes for the unfinished fourth and fifth games: $\{(A,A), (A,B), (B,A), (B,B)\}$. According to the rule of winning 3 out of 5 games, A wins in the first three cases, and only B wins in the last case; therefore, the reasonable distribution of stakes should be $3/4$ for A and $1/4$ for B.

This problem was solved by Pascal, Fermat and others in the 17th century, and their correspondence is regarded as the earliest probability literature in the history of mathematics.

In addition, teachers can also put forward the "birthday problem" in class to interact with students.

Teachers can let students guess the probability that at least two people in their class have the same birthday, and then compare the results of the interaction with the analyzed results.

Theoretically, when there are 23 students in the class, the probability that at least two people have the same birthday will exceed 50%, and when there are 50 students, the probability will reach 97%. These results are quite different from our intuition, and this difference can greatly stimulate students' interest in learning this course.

2.3. Core stats principles: life-oriented teaching

In dealing with practical problems, we usually choose to use the sample mean to approximate the population mean. The principle behind this approach is the law of large numbers - the sample mean converges to the population mean in probability.

There are many forms of the law of large numbers, but the simplest one is Bernoulli's law of large numbers, which was established in 1731 and is the first paper on probability theory.

The principle of hypothesis testing is to use part of the information provided by the sample to infer the information of the population, which can be seen everywhere in life.

For example, when a lady tastes tea, she can know the situation of the whole pot of tea with just a sip, without drinking the whole pot. Introducing the teaching of hypothesis testing through such an interesting example can improve students' learning pleasure and enthusiasm.

Letting students understand such background knowledge in class and summing up the research ideas and methods of previous mathematicians is conducive to improving students' interest, deepening their impression of the learned knowledge, and preventing students from having a "seeing trees but not the forest" feeling about the learned knowledge.

3. Combining blackboard writing with multimedia teaching

3.1. Blackboard teaching limits & multimedia advantages

The traditional teaching method of mathematics courses takes a lot of time due to the large amount of blackboard writing and slow writing speed, so the knowledge that can be taught in limited class hours is limited, and the effect is also affected.

The introduction of multimedia-assisted teaching, using projectors, computers, networks and other means, can well solve the problem of too much time spent on blackboard writing. This teaching method can combine dynamic and static, be intuitive and vivid, broaden knowledge, increase information volume, and improve learning efficiency.

In addition to the advantage of processing text, the greater advantage of multimedia is that it can intuitively and vividly present tables, images, animations and other content, highlighting the visual effect.

3.2. Multimedia shortcomings & integration with blackboard

However, in the teaching of mathematics courses, multimedia teaching cannot completely replace the role of blackboard writing.

Although multimedia teaching has many advantages, students' attention time to each PPT is extremely short. If a PPT stays for too long, students will have visual fatigue and lose interest; if the stay time is too short, students will not have time to understand and digest, leading to a chain reaction of failing to keep up with subsequent knowledge, which will be counterproductive.

Therefore, in teaching, teachers should combine blackboard writing with multimedia, and must not emphasize one at the expense of the other, so as to achieve better teaching results.

Multimedia teaching can save the time of blackboard writing, while blackboard writing makes it easier for students to keep up with the teacher's rhythm and thinking, and leaves a deeper impression.

Therefore, in teaching, the relevant definitions, concepts and other textual narrations can be presented by multimedia, while the explanation of important knowledge points, theorems and typical examples must be derived and calculated by blackboard writing. In this way, students can not only understand important knowledge points but also expand more teaching content and enrich the class.

4. Heuristic teaching

Teaching is to make students love learning and know how to learn through teachers' work. The enthusiasm of students' learning is particularly important, and the key of heuristic teaching is to mobilize students' learning enthusiasm and stimulate their desire for knowledge. It is better to teach a man to fish than to give him fish. According to the teaching purpose, content, students' knowledge level and knowledge rules, various teaching means are used to present new content through the knowledge that students are already familiar with, and connect with what they have seen in life to learn actively. This is not only a teaching method but also an idea. In the teaching of probability statistics, methods commonly used in mathematics such as analogy and scenario can be adopted to help understanding and memory.

For example, the first two simple cases of probability calculation introduced in the probability course are classical probability model and geometric probability model. Let the sample space be Ω and a random event be A . For the classical probability model, the number of sample points contained in the sample space is finite, and each sample point occurs equally likely. At this time, the probability of a random event becomes a counting problem. The probability calculation is $P(A) = \#A / \#\Omega$, where $\#A$ and $\#\Omega$ represent the number of sample points contained in A and Ω respectively. For the geometric probability model, the probability calculation is $P(A) = \mu(A) / \mu(\Omega)$, where $\mu(A)$ and $\mu(\Omega)$ are the measures (length, area, volume, etc.) of A and Ω respectively. It can be seen that these two cases are unified from the perspective of measure in advanced probability theory, except that the former is discrete counting measure and the latter

is continuous measure. Although teachers should not teach measure knowledge when lecturing, they can give students some inspiration to let them think about the connection between the two and stimulate horizontal thinking. The classical probability model cannot handle infinite possible results, but if we encounter a random event with infinitely many possible results, how should the corresponding probability be handled and calculated? How to modify the probability calculation formula to be more appropriate? After putting forward these problems, the teacher explains the types of problems handled by the geometric probability model and its application conditions, and compares them with the conditions of the classical probability model. Then students will easily understand the differences and connections between the two.

In addition, using scenarios can make abstract knowledge more vivid and easier to understand and remember. Among discrete random variables, there is a hypergeometric distribution. Its probability function is relatively complex and particularly difficult for beginners to remember. If we consider such a specific example: in a box containing N balls of two colors, red and white, there are M red balls; if we randomly select n balls from it, the probability that exactly k of the n balls are red is $C(M,k) \times C(N-M,n-k) / C(N,n)$, where $k = 0, 1, \dots, \min(n, M)$. This is the probability function of the hypergeometric distribution. In this way, we can understand an extremely abstract probability function through a simple and vivid example.

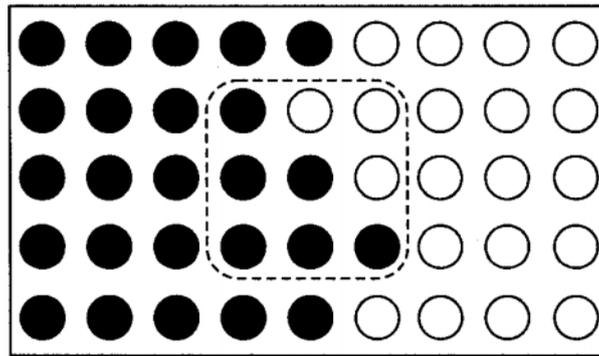


Figure 1: Example of Hypergeometric Distribution
The dashed-line area indicates that each selection of n balls contains k gray balls.

5. Using auxiliary software for teaching

The purpose of teaching probability statistics is not to let students pass exams and obtain credits, but to cultivate their ability to solve practical problems with the learned knowledge. In practical situations, what we often encounter are measured or statistical data, so the ability to directly process data is extremely important for students. Using statistical software such as R language, SPSS, SAS, and MATLAB can process a large amount of data, which also makes the teaching of probability statistics more vivid and understandable. Teachers can use software to intuitively display various density function curves and the changes of density functions with parameters. For example, teachers can use software to show the characteristics and changes of t-distribution curves with different degrees of freedom N , and display them simultaneously with the graph of the standard normal distribution. In this way, students can intuitively see that when the degree of freedom N tends to infinity, the limit distribution of the t-distribution is the standard normal distribution. At the same time, statistical problems such as interval estimation, hypothesis testing, and regression analysis can be specifically implemented through software operations. Through experimental learning and proficient use of various software, students can deepen their understanding of the taught knowledge at a deeper level, cultivate their ability to think independently and solve practical problems, and also enhance their computer application ability that should be possessed in the information age.

Stimulating students' learning interest, improving teaching effects, enabling students to learn independently and apply what they have learned is a direction that college teachers need to continuously explore. The road ahead is long and far-reaching; I will search high and low. This is an arduous and long

journey that requires college teachers to keep pace with the times, continuously innovate and explore, and constantly improve. Regarding the reform and exploration of probability statistics teaching methods, the author has talked about some superficial views in this paper and will further explore and improve them in future teaching.

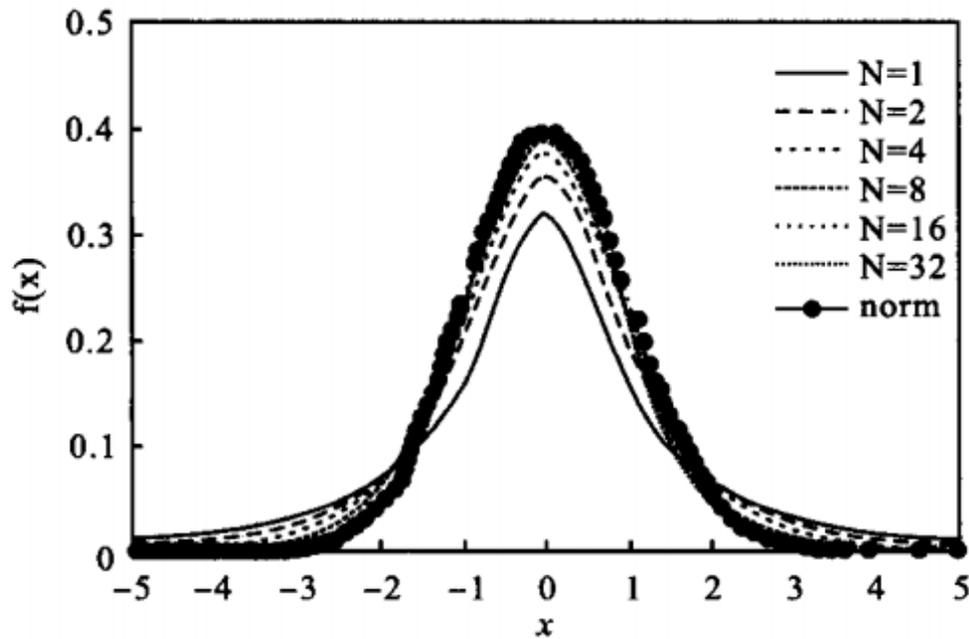


Figure 2 Density functions of the distribution at different degrees of freedom and normal distribution graph

6. Conclusion

This paper addresses the core challenges in teaching Probability Theory and Mathematical Statistics and proposes a multi-dimensional reform pathway featuring interest-driven guidance, the synergy between blackboard instruction and multimedia tools, heuristic teaching as the core, and software-assisted support. These measures closely align with the course's essential characteristic of "valuing both theoretical rigor and practical applicability": they not only resolve the difficulties posed by the abstract nature of stochastic mathematics and students' comprehension barriers but also strengthen the organic unity of knowledge transmission and competency development.

Practice has demonstrated that scientific innovation in teaching models effectively enhances students' mathematical thinking literacy and practical problem-solving abilities, which aligns with the core requirements of higher education for talent cultivation in basic disciplines. Moving forward, the teaching of Probability Theory and Mathematical Statistics must continue to keep pace with disciplinary advancements and social needs. It should be continuously optimized through balancing theoretical depth and application scope, thereby providing a solid mathematical foundation for talent cultivation across various academic fields.

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