

# On the establishment of the special theory of relativity

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**Abstract:** This article elaborates on the foundational elements of Einstein's establishment of the Special Theory of Relativity, primarily focusing on Maxwell's electromagnetic theory, the Michelson-Morley experiment, and the Lorentz transformation. Building on this, it further explores Einstein's own descriptions of the theory's development through his papers, letters, and lecture reports, aiming to deepen the understanding of Einstein's Special Theory of Relativity.

**Keywords:** Lorentz transformation, Michelson interferometer, Maxwell's equations, constancy of the speed of light

## 1. Introduction

It has been over a century since the birth of the Special Theory of Relativity, yet to this day, people's understanding of it varies significantly. For most individuals, discussing the Special Theory of Relativity is akin to talking about extraterrestrials. Upon its emergence, the Special Theory of Relativity overturned humanity's previous understanding of the physical world, leading many to perceive it as mysterious, incomprehensible, or difficult to grasp. In reality, the Special Theory of Relativity is a theory that provides a more accurate explanation of the objective physical world—a revision of the classical understanding of time and space. Before Einstein proposed the Special Theory of Relativity, Lorentz had already formulated the Lorentz coordinate transformations. However, constrained by the prevailing understanding of the "ether" at the time, the Lorentz transformations lacked a corresponding physical interpretation. Maxwell's equations did not exhibit covariance under Galilean transformations, leading to the introduction of the ether as a standard reference frame in electromagnetic theory. In 1887, the results of the Michelson-Morley experiment defied all expectations, as it failed to prove the existence of the ether. Physics, as a precise science built upon experimental foundations, was thrown into turmoil by this contradiction. People resorted to various methods and theories to explain this discrepancy.

This paper uses theories or experiments that significantly influenced Einstein's establishment of the Special Theory of Relativity as a starting point to understand and discuss the process of its development. It is hoped that this approach will enable a clearer understanding of the Special Theory of Relativity for a broader audience. Below, we will elaborate on the roles of electromagnetic field theory, the Michelson-Morley experiment, and the Lorentz transformations in Einstein's establishment of the Special Theory of Relativity, as well as Einstein's own descriptions of the process.

## 2. Maxwell's electromagnetic theory

In 1785, Coulomb formulated Coulomb's law, which describes the force between two point charges. In 1820, Oersted discovered the magnetic effect of electric currents, thereby linking electricity and magnetism. Ampère studied the interactions between currents, proposing the molecular current hypothesis and Ampère's circuital law. After Oersted's discovery of the magnetic effect of electric currents, Faraday, after more than a decade of arduous exploration, published the law of electromagnetic induction in 1831. Maxwell's outstanding contribution lay in unifying electromagnetic field theory with elegant mathematical formulations, establishing Maxwell's equations. However, within the edifice of classical physics, Maxwell's equations also exhibited certain incompatibilities.

## 2.1. The speed of light in a vacuum is a constant

The wave equation for electromagnetic waves can be derived from Maxwell's equations.

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2}$$

Since light propagates along the x-axis in a vacuum without any change in energy, we have  $dE = 0$ , where E represents the energy of the electromagnetic wave.

From

$$dE = \frac{\partial E}{\partial x} dx + \frac{\partial E}{\partial t} dt = 0$$

it can be derived that

$$c \frac{\partial E}{\partial x} = \frac{\partial E}{\partial t} \tag{1}$$

Taking the partial derivative of both sides of Equation (1) with respect to x

$$c \frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x} \left( \frac{\partial E}{\partial t} \right) \tag{2}$$

Taking the partial derivative of both sides of Equation (1) with respect to t

$$c \frac{\partial}{\partial t} \left( \frac{\partial E}{\partial x} \right) = -\frac{\partial^2 E}{\partial t^2} \tag{3}$$

Since

$$\frac{\partial}{\partial t} \left( \frac{\partial E}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{\partial E}{\partial t} \right)$$

Combining Equation (2) and Equation (3) yields

$$c \frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} \frac{1}{c} = 0$$

That is,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2} \tag{4}$$

Comparing Equation (4) with the Maxwell wave equation, it is not difficult to deduce that  $c^2 = \frac{1}{\epsilon_0 \mu_0}$ , where  $\epsilon_0$  denotes the vacuum permittivity and  $\mu_0$  denotes the vacuum permeability. Both  $\epsilon_0$  and  $\mu_0$  have been experimentally determined and are universal constants. To be precise, c here represents the propagation speed of electromagnetic waves of different frequencies in a vacuum. The electromagnetic wave most familiar to us is light, so for convenience, we will only discuss the propagation speed of light in a vacuum. At this point, a question arises: the speed of light in a vacuum does not depend on changes in the reference frame, which contradicts our understanding in classical physics. Next, we will discuss this contradiction.

## 2.2. Changes in velocity under galilean transformation of maxwell's equations

The Galilean transformation is the cornerstone of classical mechanics. This theory posits that time and space are independent and unrelated to the objects moving within them. The mathematical expression of the Galilean transformation is:

$$\begin{aligned}x' &= x - ut \\y' &= y \\z' &= z \\t' &= t\end{aligned}$$

Its physical meaning can be interpreted as follows: Assume there are two reference frames,  $S(O - xyz)$  and  $S'(O' - x'y'z')$ , with their coordinate axes mutually parallel and the  $x$ -axis coinciding with the  $x'$ -axis. The  $S'$  frame moves at a constant velocity  $u$  along the  $x$ -axis relative to the  $S$  frame. The moment when the two coordinate origins  $O$  and  $O'$  coincide is taken as the starting point for timing. This yields the spatiotemporal transformation relationship for the same object in the two coordinate systems [1].

If each equation is differentiated with respect to time, the velocity transformation equations are obtained:

$$\begin{aligned}v'_x &= v_x - u \\v'_y &= v_y \\v'_z &= v_z\end{aligned}$$

Suppose we emit a beam of light in a vacuum stationary frame. From electromagnetic theory, it is straightforward to know that the speed of light is  $c$ . However, for a spaceship moving in the same direction as the light propagation at a velocity  $u$ , is the speed still  $c$ ? According to the Galilean velocity transformation, the speed measured on the spaceship should be  $v' = c - u$ . Could it be that the speed of light in a vacuum is not constant? Or is there an issue with Maxwell's equations? Since Maxwell's equations of electrodynamics do not exhibit covariance under Galilean transformations, the introduction of "ether" as a standard reference frame in electromagnetic theory became necessary. This means that the speed of light is constant only relative to the reference frame of "ether," while all reference frames moving uniformly in a straight line relative to the "ether" do not possess this property [2].

## 3. Michelson- Morley experiment

Before the Special Theory of Relativity, the Galilean principle of relativity stated that the laws of mechanics are entirely equivalent in all inertial frames. The same physical process has the same mathematical representation in different inertial frames. Correspondingly, no mechanical experiment conducted within an inertial frame can determine whether that inertial frame itself is at rest or in uniform motion. In simpler terms, if you are sitting on a uniformly moving train with the windows blocked and cannot observe the scenery outside, you cannot determine whether the train is stationary or moving uniformly. However, pre-relativity physicists believed that electromagnetic laws did not obey the principle of relativity. They held that electromagnetic laws were valid only in a specific reference frame, referred to as the absolute reference frame or "ether." They believed that the existence of this absolute reference frame could be experimentally verified.

The most famous experiment for verifying the existence of ether is the Michelson-Morley experiment. In 1880, Michelson began planning an ether drift experiment using interferometric methods at the University of Berlin. However, due to excessive vibration interference, this initial attempt was unsuccessful. He subsequently moved the experiment to the basement of an observatory and completed it in 1881. The results were unexpected: the observed shift in interference fringes was much smaller than anticipated, approximately 0.004 to 0.005 fringes. Considering experimental error, it could almost be concluded that there was no fringe shift. Nevertheless, Michelson believed that his experimental accuracy was insufficient and that the experiment was not fully successful. As a result, the experiment was set aside for some time

without further progress. It was not until 1884, after Michelson attended an academic lecture by William Thomson (Lord Kelvin) and met with Lord Rayleigh, exchanging views on the 1881 experiment, that he regained confidence. Determined to continue the experiment, he collaborated with the renowned chemist Morley to further verify the existence of ether [3].

If light propagates through the ether, and its speed in the ether is constant, then an observer moving with a certain velocity relative to the ether should perceive the speed of light approaching from the front as greater than that catching up from behind. As the Earth moves relative to the ether, light from different directions should have different speeds. However, this difference is extremely small compared to the speed of light in the ether and cannot be directly measured [4]. Michelson sought to verify the existence of ether by measuring the interference effects caused by differences in the speed of light from different directions. Ultimately, the results of the Michelson-Morley experiment showed that, within the margin of error, the interference fringes did not shift by the theoretical expectation of 0.04 fringes. This result caused an uproar, as the vast majority of people, who were staunch supporters of the ether theory, refused to believe it. In 1922, Einstein mentioned in a speech at Kyoto University in Japan, "Even as a student, I was pondering this issue. At the time, I was aware of the strange results of the Michelson experiment. I quickly concluded that if we accepted the null result of the Michelson experiment as fact, then the idea of the Earth moving relative to the ether was mistaken. This was the earliest thought that led me to the Special Theory of Relativity" [5].

#### 4. Lorentz transformation

The null result of the Michelson-Morley experiment shocked many, but they still believed in the existence of ether and attempted to explain the null result through various theories. Among these, the most outstanding was the Lorentz transformation. Lorentz used mathematical formalism to explain the null result phenomenon. However, he did not abandon the notion of an absolute reference frame, nor did he discard the old view of spacetime. His understanding of physics remained confined to the scope of classical physics.

To explain the null result of the Michelson-Morley experiment, Lorentz boldly proposed the hypothesis that a ruler moving in absolute space would contract in the direction of motion. The Lorentz length contraction formula is:

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

In the formula,  $l_0$  represents the length of the ruler at rest in absolute space,  $l$  is its length in motion,  $v$  is the velocity of the ruler relative to absolute space, and  $c$  is the speed of light in a vacuum. Lorentz's theory of moving ruler contraction could explain the null result of the Michelson-Morley experiment. Although Lorentz used an elegant mathematical expression to explain this phenomenon, he still did not abandon the absolute reference frame and remained convinced of the existence of absolute space. After discussions with Poincaré, Lorentz further refined his theory, leading to what is now known as the Lorentz transformation. Regrettably, Lorentz still could not break free from the concept of an absolute reference frame.

The mathematical form of the Lorentz transformation differs from that of the Galilean transformation. The Lorentz transformation can revert to the Galilean transformation under low-speed conditions. In this sense, the Lorentz transformation is more comprehensive, encompassing both the low-speed scenarios of the Galilean transformation and the high-speed motion scenarios, as illustrated in Figure 1.

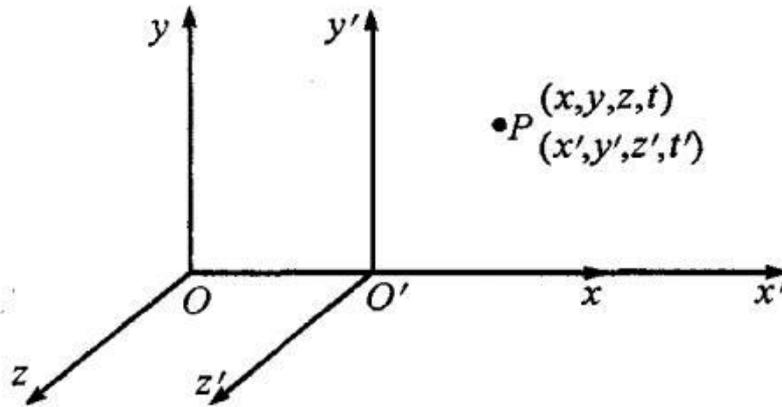


Figure 1

The mathematical form of the Lorentz transformation is:

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}} \quad y' = y$$

$$z' = z \quad t' = \frac{t - \frac{ux}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

From the coordinate transformations, the velocity transformations can be derived:

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$$

$$v'_y = \frac{v_y}{1 - \frac{uv_x}{c^2}} \sqrt{1 - \frac{u^2}{c^2}}$$

$$v'_z = \frac{v_z}{1 - \frac{uv_x}{c^2}} \sqrt{1 - \frac{u^2}{c^2}}$$

When  $u \ll c$ , the Lorentz velocity transformation reduces to the Galilean velocity transformation. When  $u$  is relatively large,  $u$  cannot be neglected. Returning to the earlier discussion of a beam of light emitted in a vacuum, we already know that the speed of light in a vacuum is  $c$ . If a spaceship moves in the same direction as the light with a speed  $u$ , according to the Lorentz transformation, substituting  $v_x = c$  yields:

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$$

The speed of light measured in the spaceship can be derived as:

$$v = \frac{c - u}{1 - \frac{uc}{c^2}} = \frac{c - u}{1 - \frac{u}{c}} = c$$

It is evident that the Lorentz transformation can not only explain the null result of the Michelson-Morley experiment but also account for the invariance of the speed of light in a vacuum as required by Maxwell's equations.

## 5. The establishment of the special theory of relativity

Einstein embraced the ideas of Austrian physicist Mach, who asserted that "concepts and statements that cannot be experimentally verified should have no place in physics" [6]. Consequently, Einstein abandoned Newton's view of spacetime, rejected the existence of ether, and sought a new spatiotemporal transformation consistent with the principle of relativity and Maxwell's electromagnetic theory. Building on the work of his predecessors, Einstein proposed two fundamental postulates: the principle of relativity and the principle of the constancy of the speed of light. Based on these two postulates, Einstein resolved the contradiction between the invariance of the speed of light and the velocity composition law, as well as issues such as asymmetries in electromagnetic theory. He also discussed the relativity of simultaneity, time dilation, length contraction, and the interplay between relativity and absoluteness. These theories provided a physical interpretation of the Lorentz transformation and offered a new understanding of spacetime.

In electromagnetism, there are two methods to generate electromotive force (EMF): induced EMF and motional EMF. Induced EMF is produced by keeping the coil stationary while changing the magnetic flux passing through it. Typically, this is achieved by moving a magnet while keeping the coil still, thereby generating an EMF. On the other hand, motional EMF is generated by moving a conductor (such as a wire) to cut through magnetic field lines. If the magnet remains stationary and the coil moves to cut through the magnetic field lines, the coil produces an EMF. Analyzing this from the perspective of relative motion, it becomes irrelevant whether the coil or the magnet moves; what matters is the relative motion between them. From this standpoint, the two methods of generating EMF are essentially the same. Einstein once discussed this in his paper, stating that Faraday's electromagnetic induction phenomenon compelled him to postulate the principle of special relativity.

Einstein held a strong belief in the correctness of his principle of relativity. Initially, he attempted to abandon the constancy of the speed of light in a vacuum, exploring the possibility of an emission theory of light. In such a theory, the speed of light remains constant only relative to its source, which would appear consistent with Einstein's principle of relativity. However, the constancy of the speed of light is a requirement of Maxwell's equations. Unless there was an issue with these equations, Einstein was inclined to trust their validity. This dilemma tormented Einstein for years. In 1924, he described how he resolved this predicament: "After seven years (1898–1905) of futile contemplation, the solution suddenly dawned on me: our concepts of space and time can claim validity only insofar as they bear a clear relationship to our experiences. Experience can indeed guide us to revise these concepts and laws. By refining the concept of simultaneity and molding it into a more flexible form, I arrived at my Special Theory of Relativity" [7]. From this, we can see that the most critical turning point in Einstein's development of the Special Theory of Relativity was his breakthrough beyond the notion of absolute spacetime, establishing a new understanding of spacetime, which ultimately resolved many of the challenges on the path to formulating the theory.

## 6. Conclusion

In a speech at Kyoto University in 1922, it was reported that Einstein once stated: "After struggling for a year with the problem of reconciling Lorentz's theory with his ideas on relativity, one day he visited a friend and discussed the issue in detail. The next day, Einstein said to his friend: 'Thank you, I have completely solved my problem.'" [7] This friend was likely Michele Besso, Einstein's colleague at the Swiss Patent Office. Besso is also the only person Einstein thanked in his paper "On the Electrodynamics of Moving Bodies." Integrating Lorentz's theory with his own ideas on relativity was not as straightforward as we might think today. It involved numerous twists and turns, ultimately leading to the birth of his Special Theory of Relativity.

## 7. References

- [1] Cheng Shouzhu, Jiang Zhiyong. General Physics. Beijing: Higher Education Press, 2008.

- [2] Zhao Xunguo. On the Physical Significance of the Lorentz Transformation. *Modern Physics Knowledge*, 2005(4).
- [3] Feng Jie. *Special Topics in University Physics*. Beijing: Peking University Press, 2011.
- [4] Wang Geng. The Michelson-Morley Experiment. *Physics Experiments*, 1984(6).
- [5] Albert Einstein. *How I Created the Theory of Relativity* (translated by Gao Xuexian, 1982.8).
- [6] Chen Dongying. The Establishment of the Special Theory of Relativity and Ideological Innovation. *Journal of Hefei University of Technology (Social Sciences Edition)*, 2005(6).
- [7] John Stachel. *Einstein's Miraculous Year: Five Papers That Changed the Face of Physics*, 2001.7.